The occurrence of losses can be attributed, following [2], to the fact that

> "if the charge motions are sufficiently rapid, then the D-E relationship must break down and show a dispersion. We therefore assume that in actuality some effect, such as a finite relaxation time for the dielectric when changing its polarization, will really prevent the wave front from ever actually achieving infinite slope, but that the relaxation time is short enough, so that the wave front can become very steep. The motion of such a steep wave front can be treated without taking into account the detailed behavior of the relaxation (or other dispersion mechanism). A treatment of the shock front, calculating its thickness as a function of the dispersion behavior, can be given [5] but is not relevant to our present purposes."

We have revised the original text by replacing the original word "ferroelectric" by "dielectric," and have changed the reference numbering.

A nonlinear dielectric cannot be a vacuum, it must contain polarizable entities, with moving charges. These moving charges must see some damping. This makes the dielectric lossy. In the simplest cases this can be represented by a series resistance, giving the capacitance the correct relaxation time for charge displacement.

Karbowiak and Freeman [4] refer to this explanation as "not tenable." Their reason: "When the resistive elements are sufficiently small, the rate of energy loss in the shock region is much too slow to account for the loss implied." It is easily shown that this statement about losses is incorrect, and that in the limit of small resistances, the loss is independent of the value of the resistance. Rather than take up the space here, for this straightforward integration, the following arguments are put forth.

1) The reader is referred to [3], and a number of its references in turn. In several of these, detailed shock structure is discussed. The shock structure is resolved, into a continuous transition with bounded slope, by any resistance in series with the nonlinear dielectric, no matter how small that resistance is. Since the equations used to derive this are consistent with conservation of energy, any energy lost in the shock propagation must be dissipated in the resistances.

2) An elementary circuit analogy is given which involves all the same physical points. Consider the discharge of a capacitor through a conductor. All of the energy originally in the capacitor is dissipated in the resistance of the conductor, no matter how small that resistance is. Consider the expression for the energy loss, $\int i^2 R dt$, as we let R become small. The length of time over which we have appreciable losses then becomes similarly small. On the other hand, i^2 goes up, during this time, as $1/R^2$. As a result, the integral remains constant.

What if in 2) we really insist on a strictly lossless conductor? Then, of course, the inductance of the current flow path cannot be neglected. The capacitive energy becomes inductive, and we establish oscillations.

What happens in our shock problem if we similarly insist on a polarizable dielectric, which (in contradiction to the dispersion relations) is genuinely lossless? Then the inertia of the charges which are displaced to establish the polarization cannot be neglected. Then, as was shown in [5], the shock does not have a simple monotonic transition, but instead oscillations are left in its wake.

Karbowiak and Freeman [4], after criticizing this author for invoking a lossy dielectric, come to the conclusion that, "It is impossible to realize a continuous loss-free transmission medium which would be characterized by (a) non-linear . . . $C(V)$." Since nonlinear dielectrics certainly exist, the conclusion must be that they are lossy. But is that not the very point they found objectionable in my work?

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Rebuttal of "Dispersion of Nonlinear Elements as a Source of Electromagnetic Shock Structure"

A. E. KARBOWIAK AND R. H. FREEMAN

***Abstract*—It is pointed out that if the classical method of weak solution is to be used for the solution of the problem, then it is necessary to include a resistive element of a sufficient magnitude. This also is a feature of Landauer's work.[1] The solution so obtained is accurate under well-defined conditions, and among others, it can be shown that energy losses associated with the shock front can be accounted for by that resistance. However, it is inconsequential *to assume* that as the value of the resistive element is reduced to zero, the energy balance continues to hold. This requires a separate proof.**

An exact analysis based on a series of experimental results and computer modeling shows that the classical discrepancy can be accounted for in a different way.

We do not think there is anything objectionable in Landauer's work,[1] but the reader should be aware that some of the observations and conclusions reached in Landauer's work are inconsequential and misleading.

At the outset, it should be clarified that a waveguiding structure can be dispersive for two distinct reasons.

1) The structure is iterative, that is, it consists of lumped parameter elements.

2) It is distributed, but with R, L, G, and C parameters such that the ratio of $R/L \neq G/C$.

In the first case the physical system can be correctly modeled by a difference equation (DE). This case, for nonlinear elements, is the subject of a separate study [1], while the other case was the subject of a recent publication (footnote 1, [4]) and is also the subject of the present discussion.

Case 2) can be modeled by partial differential equations (PDE) derivable from Maxwell's equations. Implicit in such modeling are the constitutive relations describing the equivalent line parameters (R, L, G, and C). However, energy conservation need not be obeyed. This was first commented on by Rayleigh in 1910 [2], who evidently was also puzzled by the anomaly when he said, in relation to a loss-free system, "I fail to understand how a loss of energy can be ad-

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The authors are with the School of Electrical Engineering, University of New South Wales, Kensington, N. S. W., Australia.
[1] R. Landauer, *IEEE Trans. Microwave Theory Tech.*, this issue, pp. 452–453.

mitted in a motion which is supposed to be subject to the isothermal or adiabatic law, in which no dissipative action is contemplated."

Since that time many other researchers have studied this problem, but it was Lax [3] who discussed explicitly the conservation laws, while a broad description of the problem can be found elsewhere, e.g., [4]. These studies indicate that whether the system is linear or not, Maxwell's equations *per se* are not sufficient to ensure conservation of energy and other relevant quantities, and that entropy must also be taken into account. The mathematical theory of the relevant nonlinear PDE. has been developed (particularly in the field of fluid dynamics) into a useful method, through the concept of "weak solution" [5]. Yet the theory is not entirely free from internal inconsistencies,[2] but it does enable one to obtain solutions in agreement with observations relating to viscous fluids. However, electrical transmission lines can be made with very small losses, and the weak-solution approach leaves much to be desired.

The work of Landauer used, essentially, the results relating to the weak solution. For this reason it is necessary to include resistance in the circuit to satisfy the conditions for the particular mathematical model, but this has nothing to do with the physics of the problem. Moreover, this resistance must be larger than a certain minimum value. This is a feature of the weak solution, and, as such, does not detract from the value of Landauer's contribution. But the weak solution cannot be used to deduce the correct result for a lossless case.

It is inconsequential to say that the lost energy *can* be accounted for by such ad hoc means as radiation or resistive losses. One needs to prove it, and the literature does not provide such a proof except when conditions for a weak solution hold.

Our studies based on detailed calculations, laboratory experiments, and computer modeling [1] show that shock-wave propagation in a loss-free line (which can be dispersive if of class 1) is not accompanied by energy losses, and that for a line with small losses, a complete balance of energy also holds. But this case cannot be treated by classical methods using the weak-solution concept. More specifically, one could not account for the classical energy loss associated with a shock front using the concept of weak solution, because the time constant associated with energy dissipation is too short to ensure a detailed balance of energy. Landauer's viewpoint on this matter is not, therefore, tenable.

In (footnote 1, [2]), it is stated without proof that energy losses associated with a shock front can be accounted for by resistive elements, but the reader is left wondering what happens when the system is loss free.

In the present letter, Landauer gives further material for argument by considering discharge of a condenser through a conductor. This argument is misleading. The problem so stated is improperly formulated: the mathematical model *must* contain capacitance, resistance, and inductance. One can, in principle, balance out the resistance by the suitable addition of negative resistance and therefore consider a loss-free case, and even consider a model with a negative resistance. But one may not leave out the inductance or the capacitance from the mathematical modeling.

With the correct model one can show that as the resistance is reduced to zero, so the time taken to dissipate a given amount of energy increases without limit to infinity; the same would happen with a nonlinear lumped parameter line [1]. And it is precisely for this reason that it is not good enough to assume that resistive elements *per se* can account for the energy discrepancy associated with the classical weak solution, and worse still to assume that the energy balance would also be satisfied as the resistive elements are reduced to zero.

We cannot, therefore, agree with the reasoning contained in arguments 1) and 2) of Landauer's letter.

When dealing with a distributed system, there is a further complication in that one would need to justify the step in which one passes from the DE to the corresponding PDE. For a linear problem this can be justified, but not so readily for the nonlinear case. In this latter event, and for the loss-free case, the only limit that is justifiable is the one corresponding to the linear case. Thus the conclusions concerning the realizability as stated in our publication (footnote 1, [4]) stand.

[2] For example, the method ensures conservation of momentum and energy. However, in application, energy transformation is involved and one then evokes the heat-balance equation which predicts an increase in entropy, and, as Laudau and Liftshitz [4] state, "an increase in entropy signifies energy is dissipated," wheras the shock-wave theory was specifically devised to conserve energy while permitting entropy to increase.

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Comments on "Characterization of Microwave Oscillator and Amplifier Circuits Using an IMPATT Diode Biased Below Breakdown"

N. D. KENYON

In the above short paper,[1] the method described for the characterization of IMPATT's and their circuits is based entirely upon an assumption that the whole oscillator can be correctly described by a single-resonant circuit. This the authors have been careful to emphasize. But it is by no means clear that such an assumption is tenable for any normal circuit configurations, nor that the test described to confirm the given equivalent circuit is sufficiently stringent. Though the resonant absorption may be fairly narrow, and its variation with diode bias smooth, this is no guarantee that the circuit is single tuned, that it does not, for example, require a further series reactance giving a broad resonance elsewhere, or that the components of the equivalent circuit are not themselves functions of frequency.

A case in point is the very circuit cited,[1] the resonant-cap circuit much used in millimeter-wave IMPATT evaluation. It is known [1] that this circuit has a series inductance associated with the pos-supporting the cap, and that therefore changes in absorption fret quency with diode capacitance are not so simply related as the single-tuned theory implies.

Furthermore, it is claimed[1] that other evaluation methods are handicapped by the package transformation, and yet the very existence of package parasitics makes the given equivalent circuit invalid. When a similar method [2] was applied to a circuit known to exhibit basically single-tuned properties, it was still found neces-

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The author is with the Telecommunications Headquarters, Post Office Research Department R7.2.1., Martlesham Heath, Ipswich, England IP5 7RE.

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